All relations hold for total boolean functions.
If $A$ and $B$ are connected and $A$ is below $B$, then $A = \widetilde{O}(B)$.
All listed complexity measures are $\Theta(n)$ for the parity function.
All listed complexity measures are invariant under negation.
All solid black connections hold for all total boolean functions.
All dashed red connections hold for at least one total boolean function.
All green complexity measures are not (known to be) polynomially related to the rest.

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