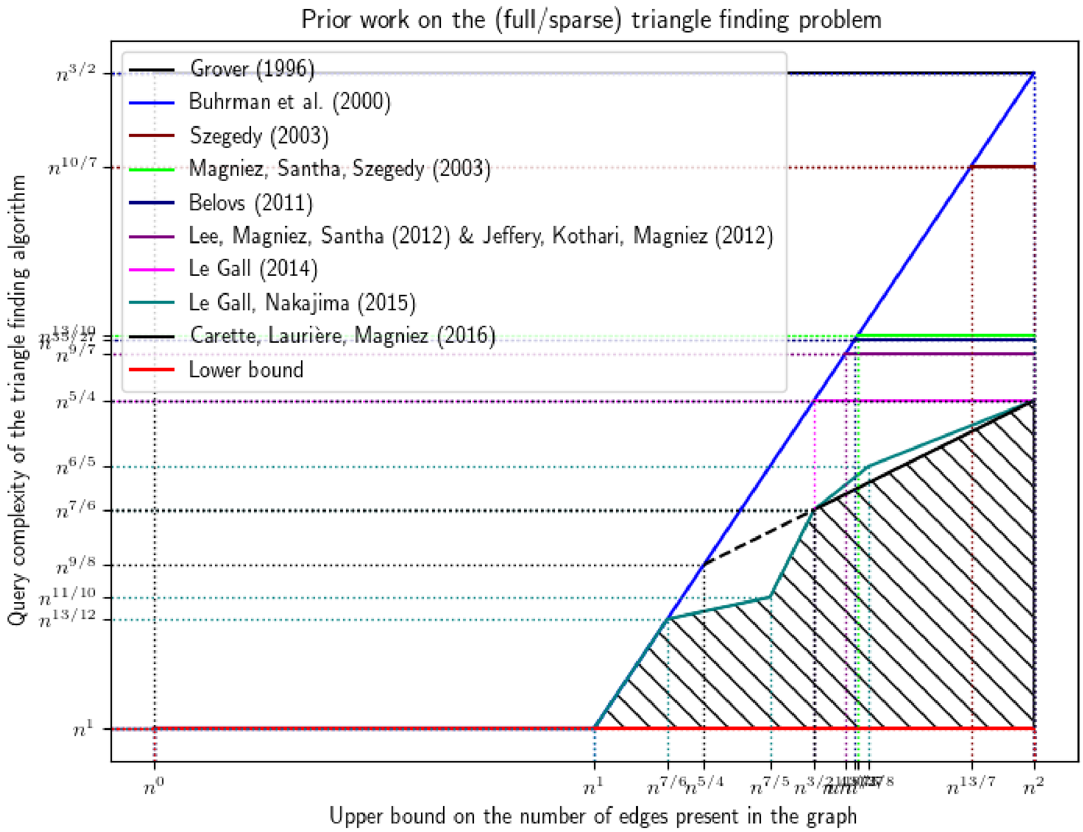

Figure 1: Graphical depiction of the historical overview of triangle finding algorithms

Author(s)	Year	Title	Query complexity
Szegedy	2003	On the query complexity of finding triangles in graphs	$\widetilde{\mathcal{O}}(n^{10/7})$
Magniez, Santha, Szegedy	2003	Algorithms for quantum triangle finding	$\widetilde{\mathcal{O}}(n^{13/10})$
Belovs	2011	Span programs for functions with constant 1-sized certificates	$\widetilde{\mathcal{O}}(n^{35/27})$
Lee, Magniez, Santha	2012	Improved quantum query algorithms for triangle finding and associativity testing	$\widetilde{\mathcal{O}}(n^{9/7})$
Jerrey, Kothari, Magniez	2012	Nested quantum walks with quantum data structures	$\widetilde{\mathcal{O}}(n^{9/7})$
Le Gall	2014	Improved quantum algorithm for triangle finding via combinatorial arguments	$\widetilde{\mathcal{O}}(n^{5/4})$

Table 1: Historical overview of algorithms for *full* triangle finding, i.e., without assumptions on the number of edges.

Author(s)	Year	Title	Query complexity
Buhrman, Dürr, Heiligman, Høyer, Magniez, Santha, de Wolf	2000	Quantum algorithms for element distinctness	$\mathcal{O}(n+\sqrt{nm})$
Le Gall, Nakajima	2015	Quantum triangle finding in sparse graphs	$\begin{cases} \mathcal{O}(n+\sqrt{nm}), & \text{if } 0 \leq m \leq n^{7/6} \\ \widetilde{\mathcal{O}}(nm^{1/14}), & \text{if } n^{7/6} \leq m \leq n^{7/5} \\ \widetilde{\mathcal{O}}(n^{1/6}m^{2/3}), & \text{if } n^{7/5} \leq m \leq n^{3/2} \\ \widetilde{\mathcal{O}}(n^{23/30}m^{4/15}), & \text{if } n^{3/2} \leq m \leq n^{13/8} \\ \widetilde{\mathcal{O}}(n^{59/60}m^{2/15}), & \text{if } n^{13/8} \leq m \leq n^{2} \end{cases}$
Carette, Laurière, Magniez	2016	Extended learning graphs for triangle finding	$\widetilde{\mathcal{O}}(n^{11/12}m^{1/6})$, if $m \geq n^{5/4}$

Table 2: Historical overview of algorithms for *sparse* triangle finding, i.e., with assumptions on the number of edges.

Author(s)	Year	Title	Parameter	Query complexity
Magniez, Santha, Szegedy	2003	Algorithms for quantum triangle finding	None	$\mathcal{O}(n^{2/3})$
Jeffery, Kothari, Magniez	2012	Improving quantum query complexity of boolean matrix multiplication using graph collision	Number of non-edges in the graph, ℓ	$\widetilde{\mathcal{O}}(\sqrt{n}+\sqrt{\ell})$
Belovs	2012	Learning graph-based quantum algorithm for k-distinctness	Size of the largest independent set, α	$\mathcal{O}(\sqrt{n}\alpha^{1/6})$
Gavinsky, Ito	2012	A quantum query algorithm for the graph collision problem	Maximum total degree of any independent set, α^*	$\widetilde{\mathcal{O}}(\sqrt{n}+\sqrt{\alpha^*})$
			Random graphs with each edge independently being present with fixed probability	$\widetilde{\mathcal{O}}(\sqrt{n})$
Ambainis, Balodis, Iraids, Ozols, Smotrovs	2013	Parametrized quantum query complexity of graph collision	Treewidth, t	$\mathcal{O}(\sqrt{n}t^{1/6})$
			α^{**} – definition below	$\mathcal{O}(\sqrt{n}+\sqrt{\alpha^{**}})$

Table 3: Overview of (parametrized) algorithms for graph collision, mainly based on Ambainis' survey presented in the paper stated above.

$$\alpha^{**} = \min_{\substack{VC\subseteq V \\ VC-\text{vertex cover of } G}} \max_{\substack{I\subseteq VC \\ I-\text{independent set}}} \sum_{v\in I} \deg(v)$$